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BALLISTIC RESEARCH LABORATORIES

REPORT No. 685

The Effect of Wind and Rotation of the Earth on Unguided Rockets

J. V. LEWIS

PROJECT NO. TB3-0608D OF THE RESEARCH AND DEVELOPMENT DIVISION, ORDNANCE DEPARTMENT

MARCH 1949

ABERDEEN PROVING GROUND, MARYLAND

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BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MARYLAND

14 April 1953

ERRATA SHEET FOR BRL REPORT NO. 685

"The Effect of Wind and Rotation of the Earth on Unguided Rockets"

Page 14 - The point of fall with respect to the launching point for zero wind with launcher vertical should read:

3700 ft. South
14,400 ft. West

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DUPLICATING BRANCH
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BALLISTIC RESEARCH LABORATORIES

REPORT NO. 685

J. V. Lewis
Aberdeen Proving Ground, Md.
6 October 1948

THE EFFECT OF WIND AND ROTATION OF THE EARTH ON UNGUIDED ROCKETS

ABSTRACT

A brief discussion is given of the determination, by weighting factor curves, of first order wind effects. Formulas for the displacement of the point of fall of an unguided rocket are given which require only the vertical velocity-time function of the rocket. The effect of tilting the launcher is given. The wind and tilt effects may be computed independently provided only that the vertical velocity-time function for the particular tilt is used. An example is given of the calculation of wind effects. A discussion is included of the effects of the rotation of the earth.

THE EFFECT OF WIND AND ROTATION OF THE EARTH ON UNGUIDED ROCKETS[1]

A. WIND EFFECTS AND WEIGHTING FACTOR CURVES

A brief summary is given here of the usual method of determining first order effects of the wind on the trajectory of a projectile.

The wind velocity is ordinarily a function of altitude. However in order to simplify the calculation of wind effect for the operators, a constant ballistic wind is used together with a unit wind effect.

Definition. The ballistic wind is that constant wind which has the same first order wind effect as the actual wind.

Definition. The unit wind effect is the wind effect of a unit constant wind.

The effect of the actual wind is then the product of the ballistic wind and the unit wind effect, assuming that the effect is proportional to the wind.

In order to compute the ballistic wind the atmosphere is divided into horizontal strata in which the wind is assumed constant and use is made of a wind weighting factor curve.

Definition. The graph of f is a wind weighting factor curve whenever f is such a function that for each positive y, f (y) is the ratio of the wind effect for a unit uniform wind up to altitude y (and zero above y) to the unit wind effect. f (y) is called the wind weighting factor for altitude y.

We make the assumptions that

(i) the effect of the wind in any particular stratum is proportional to the wind in that stratum and that

(ii) the effects of the wind in the various strata are independent.

In the next section it is shown that the above are satisfied for the displacement of the point of fall.

Under these conditions the effect of the constant wind in a single stratum is the product of the wind velocity, the unit wind effect, and the weighting factor for the stratum which is the difference of the wind weighting factors for the upper and lower altitude bounds of the stratum. Suppose the atmosphere is divided into strata with winds $w_1, w_2, \dots, w_n$ bounded by altitudes $y_0, y_1, \dots, y_n$. Let f be the wind weighting factor curve and δ be the unit wind effect. The wind effect is then given by

$$\left\{ w_1 \left[f(y_1) - f(y_0) \right] + w_2 \left[f(y_2) - f(y_1) \right] + \dots + w_n \left[f(y_n) - f(y_{n-1}) \right] \right\} \delta$$

By taking the strata sufficiently thin any desired accuracy may be obtained. The quantity in the braces is that constant wind which would have the same effect as the actual wind. Thus we have the

[1] This problem was suggested by R. H. Kent who did considerable work on it.
A treatment of the first part of this problem has been given by D. T. Perkins, "The Effect of Wind on Rockets", Air Weather Service, August 1947. The theory given here was developed independently of Perkins paper.

Formula for Ballistic Wind.

$$w_1 \left[f(y_1) - f(y_o) \right] + w_2 \left[f(y_2) - f(y_1) \right] + \ldots + w_n \left[f(y_n) - f(y_{n-1}) \right]$$

for a wind weighting factor curve f and winds $w_1, w_2, \ldots, w_n$ in strata bounded by altitudes $y_o, y_1, \ldots, y_n$.

To calculate the wind effect from observations of the wind it is thus sufficient to know the wind weighting factor curve and the unit wind effect. In the next section we show how these latter two may be found for the displacement of the point of fall of a rocket.

B. EFFECT OF WIND ON THE DISPLACEMENT OF THE POINT OF FALL OF UNGUIDED ROCKETS

The method of determination of the displacement of the point of fall given in this section was developed for a rocket for which only the vertical velocity-time function was known. It turns out that this function is the only information required.

It is assumed that the rocket is not yawing.[2] The horizontal velocity of the rocket due to the wind may then be easily determined in a reference frame moving with the wind and a correction made for the motion of the reference frame. During the flight above the atmosphere the horizontal velocity is assumed constant. The horizontal displacement of the point of fall is obtained by integrating the horizontal velocity in a fixed reference over the entire time of flight. It will be sufficient to consider the motion of the rocket and the wind velocity projected on an arbitrary vertical plane.

We make the additional

Assumptions.

1. The only forces acting on the rocket are thrust, drag, and gravity.
2. For a given launching direction, the vertical velocity is sensibly unchanged by the wind

In general, assumption (2) will be satisfied for very large winds for nearly vertical firing and for milder winds some way from the vertical.

The following notation will be used:

$v(t)$ = vertical velocity of the rocket at time t,

$u(t)$ = horizontal velocity (in a fixed reference frame) at time t,

$\hat{u}(t)$ = horizontal velocity (in reference frame moving the wind) at time t,

w = velocity of the wind,

g = acceleration of gravity,

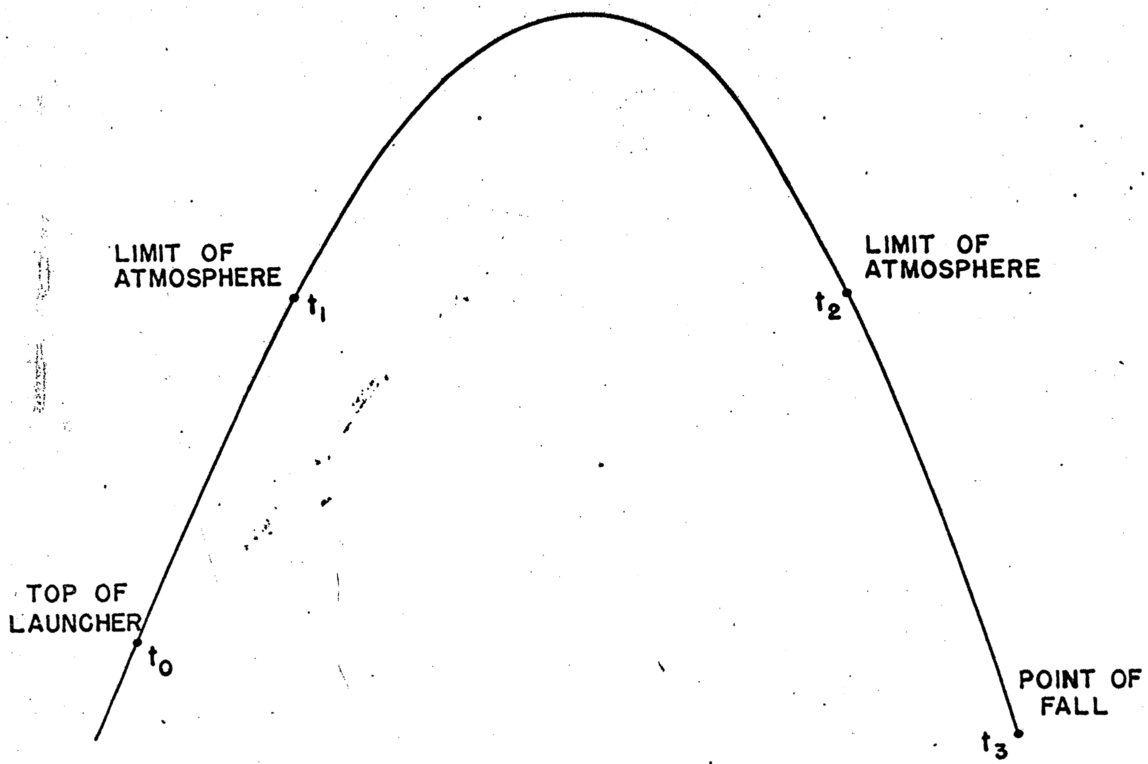
$$E(t', t'') = \operatorname{Exp} \int_{t'}^{t''} \frac{g}{v(t)}\, dt$$

for any two times t', t''.

Δ_{pq} = horizontal displacement into the wind of the point of fall caused by a wind of unit velocity in a stratum entered at time t_p and left at time t_q.

Let times t_o, t_1, t_2, t_3 divide the flight of the rocket into flight through the atmosphere and above the atmosphere as indicated in the figure. The times of entering and leaving a stratum of wind will be indicated by t_p and t_q respectively. Subscripts p, q, o, 1, 2, 3 will indicate a quantity evaluated at the corresponding time.

[2] If the rocket is yawing, a correction for the effect of yaw may be made using Ballistic Research Laboratories Report 656, "Windage Jump for Rockets Fired Nearly Vertically" by Kent and Galbraith.



For convenience of reference we state:

Condition A. The axis of the rocket is tangent of the trajectory.

Lemma.[3] If condition A is satisfied from time t_i to time t and U_i, v_i are the horizontal and velocities at time t_i, then the horizontal velocity at time t is

$$U_i \frac{v\,(t)}{v_i}\ E\,(t_i,\, t).$$

Proof. Let ξ (t) be the inclination of the trajectory from the vertical and $\mathbf{V}$ (t) be the velocity along the trajectory at time t.

Gravity is the only force with a component normal to the trajectory. Equating the centripetal acceleration normal to the trajectory to the normal component of the acceleration of gravity we obtain:

$$\mathbf{V}\,\dot{\xi} = g \sin \xi\ .$$

Using

$$v = \mathbf{V} \cos \xi$$

the above differential equation becomes

$$\frac{2\ \dot{\xi}}{\sin 2\xi} = \frac{g}{v}\ .$$

[3] This formula in a slightly different form was used by Perkins, Op. Cit.

Its integral is

$$\tan \xi(t) = \tan \xi(t_i)\, E\,(t_i, t).$$

In terms of the initial conditions this is

$$\tan \xi\,(t) = \frac{U_i}{v_i}\, E\,(t_i, t)$$

The horizontal velocity at time t, is

$$v\,(t) \tan \xi\,(t) = U_i \frac{v\,(t)}{v_i}\, E\,(t_i, t)$$

which was to be shown

EFFECT OF A CONSTANT WIND IN A HORIZONTAL STRATUM

Let the atmosphere be divided into horizontal strata. For the purpose of computing the effect of the wind in any particular stratum, we assume that the wind velocity encountered by the rocket after passing through this stratum is zero. It will be shown that the effect of this stratum of wind may be computed independently of the effects of the strata through which the rocket has already passed. It then follows that the effects of all wind strata may be computed independently.

We now consider the motion of the rocket and the wind velocity projected on an arbitrary vertical plane. Take a rectangular coordinate system in this plane with horizontal and vertical axes. Since the rocket is not yawing, condition A is satisfied in a reference frame moving with the wind. While the rocket is in the stratum, for $t_p \leq t \leq t_q$, we infer from the lemma that

$$\hat{u}\,(t) = \hat{u}_p \frac{v\,(t)}{v_p}\, E\,(t_p, t). \tag{1}$$

Using

$$u\,(t) = \hat{u}\,(t) + w \tag{2}$$

to change to the fixed reference frame we obtain

$$u\,(t) = u_p \frac{v\,(t)}{v_p}\, E\,(t_p, t) - w \frac{v\,(t)}{v_p}\, E\,(t_p, t) + w \tag{3}$$

for $t_p \leq t \leq t_q$. In particular

$$u_q = u_p \frac{v_q}{v_p}\, E\,(t_p, t_q) - w \frac{v_q}{v_p}\, E\,(t_p, t_q) + w \tag{4}$$

Condition A is valid in the fixed reference frame after the rocket has passed through this stratum since the wind velocity is zero. Thus we infer from the lemma that:

$$u(t) = u_q \frac{v(t)}{v_q} E(t_q, t) \tag{5}$$

for $t_q \leq t$. Using (4) this becomes:

$$u(t) = u_p \frac{v(t)}{v_p} E(t_p, t) - w \frac{v(t)}{v_p} E(t_p, t) + w \frac{v(t)}{v_q} E(t_q, t) \tag{6}$$

for $t_q \leq t$.

The horizontal velocity of the rocket can be divided into two parts: the part due to the wind in the stratum and the part due to tilt of the launcher and effects of wind strata through which the rocket has already passed.

Using (3) and (6) we find that at time t the horizontal velocity in the fixed reference frame due to the wind in the stratum is:

$$\begin{aligned} &0, && \text{for } t \leq t_p, \\ &w - w \frac{v(t)}{v_p} E(t_p, t), && \text{for}^4\ t_p \leq t \leq t_q, \\ &w \frac{v(t)}{v_q} E(t_q, t) - w \frac{v(t)}{v_p} E(t_p, t), && \text{for } t_q \leq t. \end{aligned} \tag{7}$$

The above velocity is taken as zero before time t_p when the rocket enters the stratum. Again using (3) and (6) we see that the horizontal velocity u_p at time t_p due to the tilt of the launcher and to effects of wind strata through which the rocket has already passed, contributes a horizontal velocity in the fixed frame which at time t is

$$u_p \frac{v(t)}{v_p} E(t_p, t), \qquad \text{for } t_p \leq t. \tag{8}$$

[4] This expression is equivalent to the well known one of ballistics

$$w - w \operatorname{Exp} \int_{t_p}^{t} E(\tau)\, d\tau$$

Here E has its usual ballistic significance. The present formula is used because only the vertical velocity is needed.

This latter velocity is unaffected by the wind in the stratum.

We can conclude that the wind effect of a particular stratum is proportional to the wind velocity in the stratum and is independent of the effects of strata through which the rocket has already passed. Consequently the effects of all wind strata may be computed independently. Furthermore the effect of tilting the launcher may be found independently (provided the vertical velocity for the particular tilt is used) since the tilt may be regarded as contributing a horizontal velocity at the top of the launcher.

The horizontal displacement of the point of fall due to the wind stratum is obtained by integrating the horizontal velocity due to this stratum in the fixed reference frame over the entire time of flight. If the wind stratum is encountered while the rocket is ascending the displacement of the point of fall will be much greater than if the wind stratum is encountered while descending since the horizontal velocity acquired has a much longer time over which to act. Upon integration of (7) (the horizontal velocity being taken as constant during the flight above the atmosphere), division by w and change of sign, the following results are obtained:

For $t_o \leq t_p < t_q \leq t_1$ (rocket ascends through stratum)

$$\Delta_{pq} = \frac{1}{v_p} \int_{t_p}^{t_1} v(t)\, E(t_p, t)\, dt - \frac{1}{v_q} \int_{t_q}^{t_1} v(t)\, E(t_q, t)\, dt - (t_q - t_p)$$

$$+ \left[E(t_p, t_1)/v_p - E(t_q, t_1)/v_q \right] \left[v_1 (t_2 - t_1) - \int_{t_2}^{t_3} v(t)\, E(t_2, t)\, dt \right] \qquad (9)$$

For $t_2 \leq t_p < t_q \leq t_3$ (rocket descends through stratum)

$$\Delta_{pq} = \frac{1}{v_p} \int_{t_p}^{t_3} v(t)\, E(t_p, t)\, dt - \frac{1}{v_q} \int_{t_q}^{t_3} v(t)\, E(t_q, t)\, dt - (t_q - t_p) \qquad (10)$$

To obtain the weighting factor curve and unit effect for the displacement of the point of fall, it is necessary for each altitude y to add the displacements (9) and (10) for a uniform wind stratum up to altitude y. The velocity v used in the computation is that for zero wind in accordance with assumption 2.

C. DISPLACEMENT OF POINT OF FALL DUE TO TILT OF LAUNCHER

The effect of tilting the launcher may be found by regarding the tilt as contributing a horizontal velocity u_o at the top of the launcher. Set $p = o$ in (8) to determine the horizontal velocity due to the tilt at any time t for $t_o \leq t$. This velocity is taken constant during the flight of the rocket above the atmosphere. Upon integration the displacement of point of fall in the direction of tilt caused by the tilt of the launcher is found to be the product of the tangent of the angle of tilt from the vertical and

$$\int_{t_o}^{t_1} v\, E\,(t_o, t)\, dt + E\,(t_o, t_1) \left[v_1\,(t_2 - t_1) - \int_{t_2}^{t_3} v\, E\,(t_2, t)\, dt \right]$$

D. ROCKETS FIRED NEARLY VERTICALLY

In this case the effect of tilting the launcher and the wind effect are independent. For we have noted that the effect of tilting the launcher and the wind effect may be computed independently so long as the vertical velocity for the particular tilt is used. In the present case this velocity remains sensibly unchanged by both the wind and tilt.

E. WIND EFFECT FOR AEROBEE FIRED NEARLY VERTICALLY

The weighting factor curve in figures 1 and 2 for the displacement of the point of fall for Aerobee fired nearly vertically has been computed under the assumption that the rocket does not yaw after the end of the booster action and that the atmosphere extends only up to the altitude of 110,000 feet at the end of burning.

We now give an example of the calculation of the displacement of the point of fall of Aerobee due to a given wind. We direct our attention first to obtaining the north component of the displacement. Suppose the north component of wind is given in the table:

Bounding Altitude of Stratum (feet)	Wind Velocity (feet/sec)
100- 300	7
300- 500	5
500- 1,000	2
1,000- 10,000	-5
10,000- 30,000	0
30,000- 50,000	22
50,000- 70,000	38
70,000- 90,000	70
90,000-110,000	45

Taking differences from figures 1 and 2 we obtain the weighting factors for the corresponding horizontal strata.

Bounding Altitudes of Stratum	Weighting Factor
100- 300	.200
300- 500	.330
500- 1,000	.124
1,000- 10,000	.176
10,000- 30,000	.100
30,000- 50,000	.038
50,000- 70,000	.015
70,000- 90,000	.009
90,000-110,000	.008

The north component of the ballistic wind (the uniform wind which will have the same effect on the point of fall as the actual wind) is found by forming the product of the wind velocity and weighting factor in the corresponding strata and summing:

$$7\ (.200) + 5\ (.330) + \ldots + 45\ (.008)$$

The product of the north component of the ballistic wind 4.81 ft/sec and the unit effect 11,840 ft (effect of a unit uniform wind) is the displacement of the point of fall to the north 57,000 ft. A similar procedure gives the displacement of the point of fall to the east.

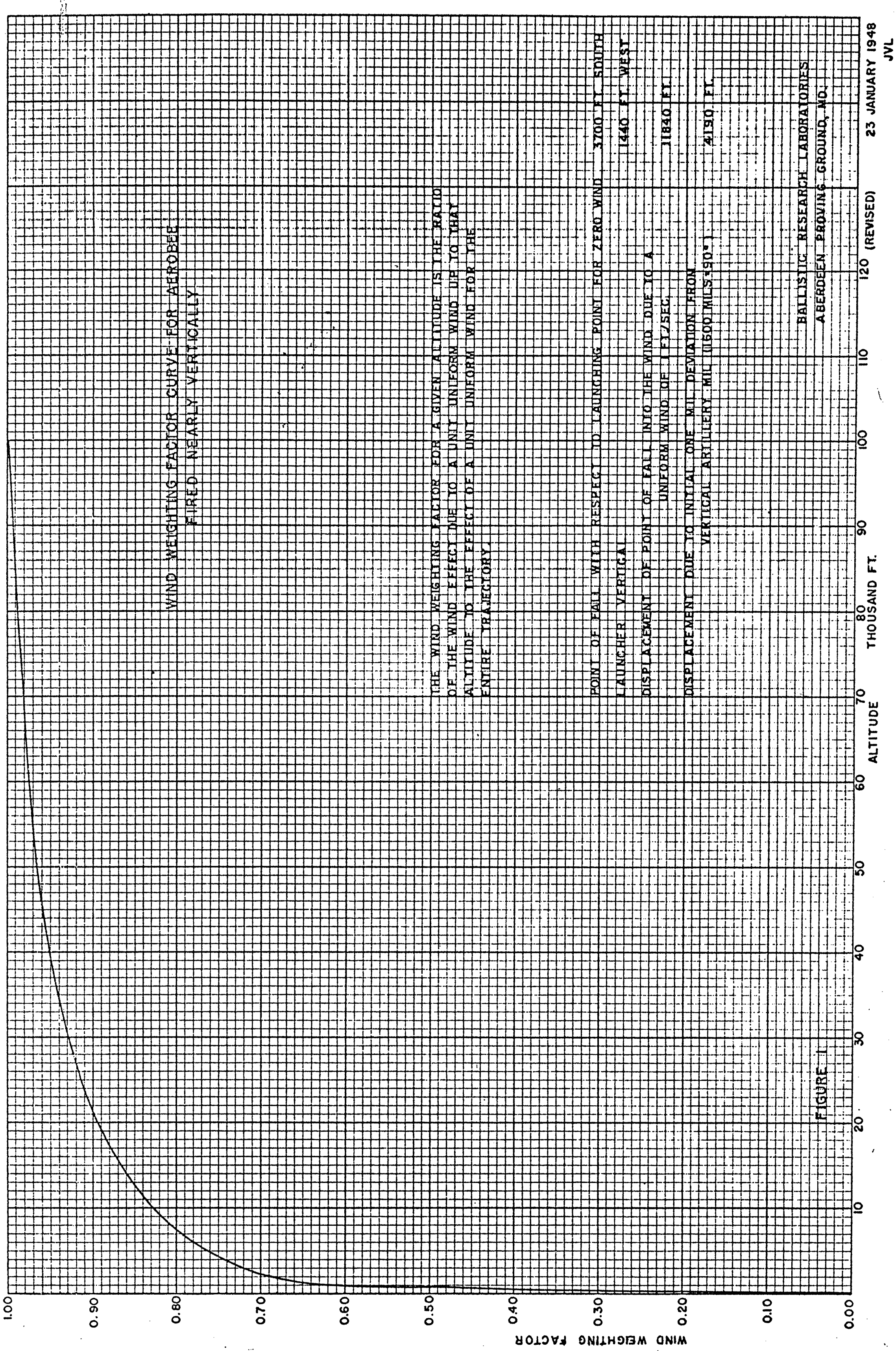

WIND WEIGHTING FACTOR CURVE FOR AEROBEE
FIRED NEARLY VERTICALLY
THE WIND WEIGHTING FACTOR FOR A GIVEN ALTITUDE IS THE RATIO
OF THE WIND EFFECT DUE TO A UNIT UNIFORM WIND UP TO THAT
ALTITUDE TO THE EFFECT OF A UNIT UNIFORM WIND FOR THE
ENTIRE TRAJECTORY.
POINT OF FALL WITH RESPECT TO LAUNCHING POINT FOR ZERO WIND 3700 FT SOUTH
1440 FT WEST
LAUNCHER VERTICAL
DISPLACEMENT OF POINT OF FALL INTO THE WIND DUE TO A
UNIFORM WIND OF 1 FT/SEC. 11840 FT.
DISPLACEMENT DUE TO INITIAL ONE MIL DEVIATION FROM
VERTICAL ARTILLERY MIL (1600 MILS=90°) 4190 FT.
BALLISTIC RESEARCH LABORATORIES
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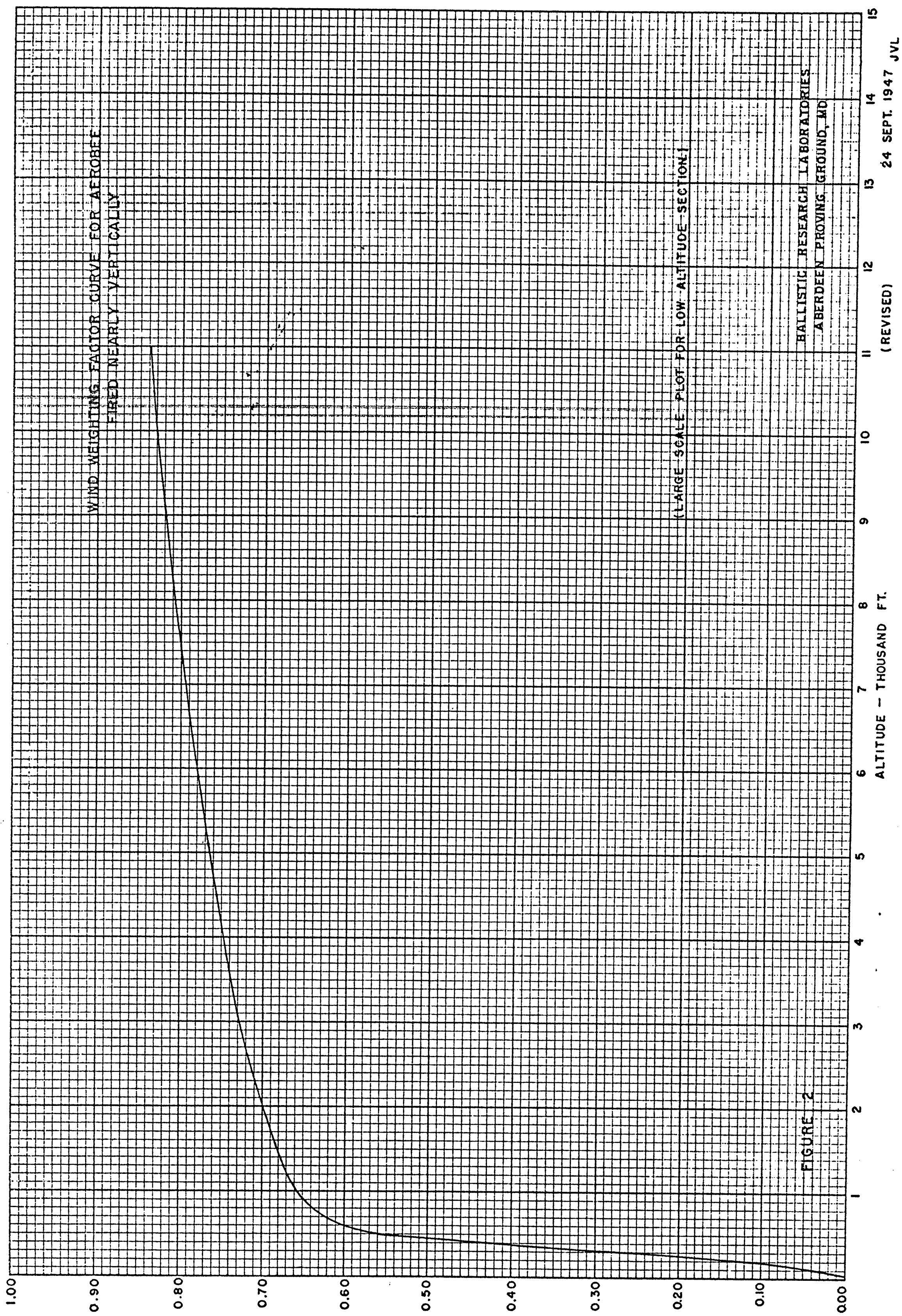
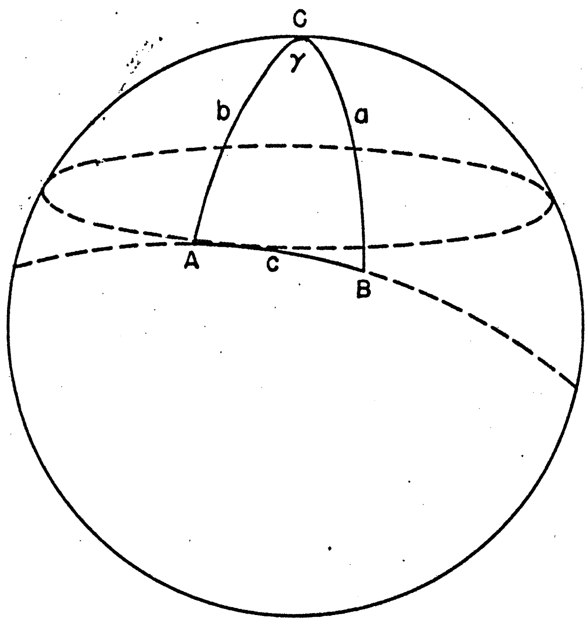
FIGURE 1.
10 20 30 40 50 60 70 80 90 100 110 120
ALTITUDE THOUSAND FT.
(REVISED) 23 JANUARY 1948
JVL
1.00 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10 0.00
WIND WEIGHTING FACTOR


WIND WEIGHTING FACTOR CURVE FOR AEROBEE
FIRED NEARLY VERTICALLY
(LARGE SCALE PLOT FOR LOW ALTITUDE SECTION)
BALLISTIC RESEARCH LABORATORIES
ABERDEEN PROVING GROUND, MD
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
ALTITUDE – THOUSAND FT.
(REVISED)
24 SEPT. 1947 JVL
FIGURE 2

F. EFFECTS OF THE ROTATION OF THE EARTH

Suppose a rocket is fired vertically[5] upward from the earth. Refer the motion to a non-rotating reference frame. The rocket is given a tangential velocity by the earth's rotation. The resultant velocity vector of the rocket at launching lies in the plane determined by the vertical and the tangent to the latitude circle at the launcher. Except for small atmospheric effects the entire trajectory of the rocket lies in this plane, through the earth's center and the tangent to the latitude circle at the launcher. The projection of the rocket's trajectory upon a non-rotating sphere about the earth's center is a great circle tangent to the latitude circle at the launcher. This is illustrated in the figure below.



where A B C is a spherical triangle and

A is the position of the launcher,
B is the point of fall,
C is the north pole,
a is the colatitude of the point of fall,
b is the colatitude of the launcher,
c is the angle subtended at the earth's center by the trajectory of the rocket
γ is the difference in longitude of launcher and point of fall.

In the spherical triangle A B C, the angle at A is a right angle and b and c are known. The angle b is fixed by the launching site, and c can be determined in various ways as will be shown. Accordingly a and γ are given by·

[5] This is the only case considered here. However, the effects obtained are a good approximation when the rocket is fired nearly vertically and the effects for the general case can be obtained with slight changes in the equations.

$$\cos a = \cos b \cos c \tag{1}$$

and

$$\sin \gamma = \frac{\sin c}{\sin a}\,. \tag{2}$$

The southerly change in latitude is

$$a - b \tag{3}$$

and the westerly change in longitude on the rotating earth is

$$\omega T - \gamma \tag{4}$$

where

ω is the angular velocity of the earth,
T is the time of flight.

Let

R_o = distance of launcher from earth's center,
r_o = distance of launcher from earth's axis.

Then the southerly displacement is

$$(a - b)\, R_o \tag{5}$$

and the westerly displacement is

$$(\omega T - \gamma)\, r_o \tag{6}$$

along the latitude circle of the launcher on the rotating earth.

It remains to determine the angle c which the trajectory subtends at the earth's center and the time T of flight.

CONSERVATION OF ANGULAR MOMENTUM

If the altitude-time function is known for the trajectory, then the time T of flight is known and the principle of conservation of angular momentum about the earth's center can be used to determine c.

Let

y = altitude of rocket above launcher,
R = distance of rocket from earth's center,
Ω = angular velocity of rocket about earth's center.

The angular momentum per unit mass about the earth's center is then $R^2\Omega$. The initial angular velocity given the rocket by the earth's rotation is $\frac{\omega r_o}{R_o}$. It follows that

$$R^2 \Omega = \omega r_o R_o,$$

$$\Omega = \frac{\omega r_o R_o}{R^2}, \tag{7}$$

$$c = \int_o^T \Omega\, dt = \omega r_o R_o \int_o^T \frac{1}{R^2}\, dt, \tag{8}$$

the time being measured from the instant the rocket leaves the launcher.

A very good approximate expression for the westerly displacement may be obtained by using (8), when c is small. In this case the latitude of the point of fall differs little from the latitude of the launcher. Using (6), (2), and (8),

$$(\omega T - \gamma)\, r_o \cong \omega\, r_o T - \frac{c}{\sin a}\, r_o$$

$$\cong \omega\, r_o T - c\, R_o$$

$$= \omega\, r_o T - \omega\, r_o R_o^2 \int_o^T \frac{1}{R^2}\, d\, t$$

$$= \omega\, r_o \int_o^T \left[1 - \left(\frac{R_o}{R} \right)^2 \right] d\, t$$

$$\cong \omega\, r_o \int_o^T \frac{2\,(R - R_o)}{R}\, dt$$

$$\cong \frac{2\,\omega\, r_o}{R_o} \int_o^T (R - R_o)\, dt$$

$$= 2\,\omega \sin b \int_o^T y\, dt.$$

Thus the westerly displacement is approximated closely by

$$2\,\omega \sin b \int_o^t y\, dt. \qquad (9)$$

(8) and (9) may be used to determine the angle subtended at the earth's center by any part of the trajectory and the corresponding westerly displacement provided the integration is taken over the corresponding time.

In the event that the altitude-time function is not known for the entire trajectory, recourse may be had to formulas based on central force theory for the flight of the projectile above the atmosphere.

FLIGHT OF ROCKET ABOVE THE ATMOSPHERE

Let initial conditions be taken at a point assumed to be above the atmosphere. It is often convenient to take these initial conditions as those at burnout. Let

v_1 = initial vertical velocity,

e_1 = initial tangential velocity due to earth's rotation,

R_1 = initial distance from earth's center,

G_1 = initial acceleration of true gravity (without centripetal acceleration),

V_1 = initial velocity in space $\sqrt{v_1^2 + e_1^2}$,

$\tan\varphi = v_1/e_1$,

$\lambda = V_1^2/R_1 G_1$.

From (7) it follows that

$$e_1 = \omega r_o \left(\frac{R_o}{R_1}\right). \tag{10}$$

According to the ordinary inverse square central force theory the rocket describes an elliptical orbit above the atmosphere. The angle subtended at the earth's center by the part of the trajectory above the atmosphere is

$$2 \arc\tan \frac{\lambda \tan\varphi}{1 - \lambda + \tan^2\varphi}. \tag{11}$$

The time of flight above the atmosphere is

$$\frac{2 R_1 \lambda}{V_1(2 - \lambda)} \left[\sin\varphi + \frac{1}{\sqrt{\lambda(2 - \lambda)}} \arc\tan \left(\frac{\sin\varphi\sqrt{\lambda(2 - \lambda)}}{1 - \lambda}\right)\right]. \tag{12}$$

The altitude of the summit above the initial point is

$$\frac{R_1}{2 - \lambda}\left[\lambda - 1 + \sqrt{1 - \lambda(2 - \lambda)\cos^2\varphi}\right]. \tag{13}$$

NOTATION

A = position of launcher

a = colatitude of point of fall

B = point of fall in non-rotating reference frame

b = colatitude of launcher

C = north pole

c = angle subtended at earth's center by the trajectory of the rocket in non-rotating reference frame.

γ = difference in longitude of launcher and point of fall in non-rotating reference frame.

Δ_{pq} = horizontal displacement into the wind of the point of fall caused by a wind of unit velocity in a stratum entered at time t_p and left at time t_q,

δ = unit wind effect

$$E(t', t'') = \mathrm{Exp} \int_{t'}^{t''} \frac{g}{v(t)} \, dt$$

e = tangential velocity due to earth's rotation,

f = wind weighting factor,

G = acceleration of true gravity (without centripetal acceleration),

g = apparent acceleration of gravity

i = subscript indicating initial value

$\lambda = V_1^2 / R_1 G_1$

ξ = inclination of trajectory from the vertical

p = subscript indicating value when rocket enters wind stratum

q = subscript indicating value when rocket leaves wind stratum

R = distance from center of earth

r = distance from earth's axis

T = time of flight

t = time

φ = arc tan v_1/e_1

U = horizontal velocity

u = horizontal velocity in fixed reference frame

$\hat{u}$ = horizontal velocity in reference frame moving with the wind,

$\mathbf{V}$ = velocity along the trajectory

V = velcoity in space

v = vertical velocity

w = wind velocity

y = altitude above launcher

Ω = angular velocity of rocket about earth's center

ω = angular velocity of earth

o = subscript indicating value at launcher,

1 = subscript indicating value when rocket leaves atmosphere

2 = subscript indicating value when rocket returns to atmosphere

3 = subscript indicating value at point of fall.